A TEMPERATURE FIELD PROBLEM

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The problem of the temperature field in bodies with heat release and with internal cooling channels in the presence of external heat transfer is solved through the introduction of the concept of sink intensity.

In performing the thermal calculations for materials with internal heat sources we frequently encounter the problem of determining the temperature field of individual elements with internal cooling channels. Here we are interested in finding the true efficiency of the cooling channel, determined by the amount of heat removed. In thermophysics such problems are usually solved by specifying the cooling channel in the form of a boundary condition of the third kind or by matching the solutions for various regions. This approach makes it possible to achieve a solution for materials [bodies] cylindrical in shape, with a coaxial channel.

In the general case, the channel may be positioned arbitrarily, and then these methods will not be suitable to find a solution for the stated problem, although an approximate idea of the temperature field can be obtained by replacing the concentrated chennel with one that is distributed as a consequence of an equivalent increase in the external heat transfer.

It is therefore held to be necessary that the calculation of the heating of such bodies is performed by specifying the channel in the form of a linear sink, situated in the center of the channel. The quantity of heat removed by such a cooling channel can be determined experimentally from the data for the superheating of the cooling medium at the ends of the channel and from the flow rate of the cooling medium. Since it is difficult to achieve such data in a number of cases, we are interested in determining the intensity of the sink in a theoretical manner, bearing in mind the true dimensions of the channel, i.e., the channel is not drawn out into a line, but is treated as a three-dimensional sink with an intensity W.

Let us consider a body (Fig. 1) in the shape of a hollow cylinder in which the intensity of the internal heat sources is a function of both r and z. The heat evolved in the body is transmitted to the external cooling medium and into the internal cooling channel. We will restrict ourselves to the problem in which the channel intensity $W = \text{const} = W_{av}$, which corresponds to the case of small channel dimensions and an insignificant tangential temperature gradient across the channel.

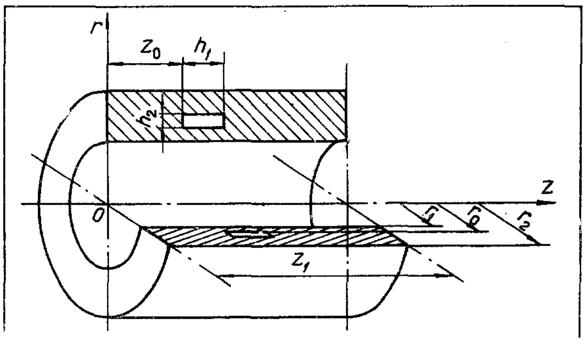

Fig. 1. Theoretical model.

The temperature field in such a body will then be determined from the following boundary-value problem:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Theta}{\partial r}\right)+\frac{\partial^2\Theta}{\partial z^2}=-\frac{P(r,\ z)}{\lambda}+\frac{1}{\lambda}\left[\eta(z-z_0)-\eta(z-z_0-h_1)\right]\times\left[\eta(r-r_0)-\eta(r-r_0-h_2)\right]\left[P(r,\ z)+W\right]; \quad (1)$$

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TABLE 1. Temperature (°C) of the Intermediate Insert of a 300 MW Turbogenerator

x, m	y, m				
	0	0,035	0,052	0,089	0,140
0	69,0	66,7	63,4	63,0	68
0,05	63,5	55,0	49,5	48,8	53,8
0,1	58,6	52,2	47,0	46,0	60,0

$$-\frac{\partial\Theta}{\partial r}+\frac{\alpha_1}{\lambda}(\Theta-\Theta_1)\Big|_{r=r_1}=0,\quad \frac{\partial\Theta}{\partial r}+\frac{\alpha_2}{\lambda}(\Theta-\Theta_2)\Big|_{r=r_2}=0; \tag{1a}$$

$$-\frac{\partial\Theta}{\partial z}+\frac{\alpha_3}{\lambda}(\Theta-\Theta_3)\Big|_{z=0}=0,\quad \frac{\partial\Theta}{\partial z}+\frac{\alpha_4}{\lambda}(\Theta-\Theta_4)\Big|_{z=z_1}=0. \tag{1b}$$

The second term in the right-hand member of (1) describes the removal of heat in the cooling channel, with consideration given to the absence of losses in the channel itself. In this case, we use the Heaviside unit η-function

$$\eta(z)=\begin{cases}1 & z\geqslant 0,\\ 0 & z<0.\end{cases} \tag{2}$$

Equation (1) with the specified boundary conditions (1b) is turned into an ordinary differential equation by applying the finite integral transform to the function $\Theta(r, z)$ with respect to the variable z with a kernel of the form

$$\cos(\beta_n z+\delta_n),\quad n=1,\ 2,\ \ldots$$

The constants β_n and δ_n are determined from the following equations:

$$\operatorname{ctg}\beta_n z_1=\frac{\lambda\beta_n-\dfrac{\alpha_3\alpha_4}{\beta_n\lambda}}{\alpha_3+\alpha_4},\quad \delta_n=-\operatorname{arctg}\frac{\alpha_3}{\beta_n\lambda}. \tag{3}$$

The solution of the boundary-value problem (1), (1a), and (1b) will depend on W. We then compile the equation for W, using Newton's formula for the channel walls, i.e.,

$$lW=\alpha[\Theta(r,\ z,\ W)-\Theta_0]. \tag{4}$$

From this expression we see that the sink intensity W, in the general case, is a function of the coordinates r and z of the channel region. However, for the case under consideration, as indicated above, we must choose the average lW for the walls. With a uniform distribution of the specific losses along r, the sink intensity is therefore determined from (4), compiled for the point $(r_0,\ z_0+h_1/2)$.

However, if P(r, z) changes markedly in the channel region along r, we have to compile yet another equation, in analogous fashion, for the point $(r_0+h_2,\ z_0+h_1/2)$ and taking the average value of W.

If the body has n channels removing heat expressed by $\sum_{i=1}^{n} W_i h_{1i} h_{2i} l_i$, we compile the following system of equations:

$$l_i W_i=\alpha_i\left[\Theta\left(r_{0i},\ z_{0i}+\frac{h_{1i}}{2},\ W_1,\ W_2,\ \ldots,\ W_n\right)-\Theta_{0i}\right], \tag{4a}$$

$$i=1,\ 2,\ \ldots,\ n.$$

Joint consideration of the boundary-value problem (1), (1a), (1b), and (4) makes it possible to determine the magnitude of the heat removed to the channel and the temperature field of the body. It would not be reasonable to present the expressions for these quantities in general form in view of their cumbersome form; we will therefore illustrate the proposed method with one of the problems of heating in electrical equipment with liquid cooling, where the rotating electromagnetic fields lead to the evolution of heat in ferromagnetic components that are symmetrical with respect to the shaft of the equipment. The specific losses within such components are independent of φ, and intensive cooling of such components, as a rule, results in only slight heating of the liquid in the channel.

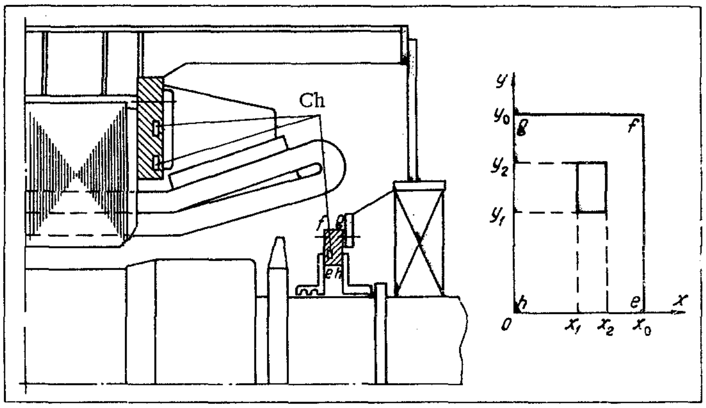

Fig. 2. End portion of a powerful turbogenerator (a) (Ch denotes channels with the cooling liquid) and a cross section of the intermediate insert (b) in the efgh plane ($y_2 - y_1 = h_2$.

The above can be referred to most of the elements in the forward zone (Fig. 2) of powerful turbogenerators for which the problem of heating is currently very important [1].

We will illustrate this method of heating calculations for bodies with internal cooling channels on the example of an element whose dimensions – to simplify the calculations – permit us to change to rectangular coordinates.

As such an example we will determine the efficiency of the cooling channel in the intermediate insert of a turbogenerator developing a power of 300 MW (Fig. 2b). The insert is insulated from the housing of the bearing along the line gh. We have intensive heat transfer from the surfaces ef and fg to the gas being cooled, while the transfer of heat from the surface eh can be neglected. The boundary conditions will then have the form

$$\left.\frac{\partial\Theta}{\partial x}+\frac{\alpha_1}{\lambda}(\Theta-\Theta_1)\right|_{x=x_0}=0,\quad \left.\frac{\partial\Theta}{\partial x}\right|_{x=0}=0,$$
$$\left.\frac{\partial\Theta}{\partial y}+\frac{\alpha_2}{\lambda}(\Theta-\Theta_2)\right|_{y=y_0}=0,\quad \left.\frac{\partial\Theta}{\partial y}\right|_{y=0}=0. \tag{5}$$

Bearing the above in mind, in a rectangular system of coordinates, we find the temperature function in the form

$$\Theta=\sum_{n=1}^{\infty}(\tilde{\Theta}_{1n}+\tilde{\Theta}_{2n})\frac{\cos\beta_n x}{\frac{x_0}{2}+\frac{1}{4\beta_n}\sin 2\beta_n x_0}+\Theta_1, \tag{6}$$

$$\Theta_{1n}=\frac{\operatorname{ch}\beta_n y}{\beta_n\operatorname{sh}\beta_n y_0+\frac{\alpha_1}{\lambda}\operatorname{ch}\beta_n y_0}\ \frac{\alpha_1\sin\beta_n x_0}{\lambda\beta_n}\left[\Theta_2-\Theta_1-\frac{P}{\lambda\beta_n^2}\right]+\frac{P}{\lambda\beta_n^3}\sin\beta_n x_0; \tag{6a}$$

$$\Theta_{2n}=(P+W)\frac{\sin\beta_n x_2-\sin\beta_n x_1}{\lambda\beta_n^3}\left\{-2\operatorname{ch}\beta_n y\operatorname{sh}\beta_n\frac{h_2}{2}\right.$$
$$\left.\times\frac{\beta_n\operatorname{ch}\beta_n\left(y_0-y_1+\frac{h_2}{2}\right)+\frac{\alpha_1}{\lambda}\operatorname{sh}\beta_n\left(y_0-y_1-\frac{h_2}{2}\right)}{\beta_n}+[\operatorname{ch}\beta_n(y-y_1)-b_n(y)]\right\}; \tag{6b}$$

$$b_n(y)=\begin{cases}\operatorname{ch}\beta_n(y-y_1) & 0\leqslant y<y_1,\\ 1 & y_1\leqslant y\leqslant y_1+h_2,\\ \operatorname{ch}\beta_n(y-y_1-h_2) & y_1+h_2<y\leqslant y_0\end{cases} \tag{6c}$$

for $\alpha_1=\alpha_2$.

The numerical values in these formulas for the insert were $P = 600\ kW/m^3$, $x_0 = 0.1$ m, $y_0 = 0.14$ m, $x_1 = 0.054$ m, $L = 1.60$ m, $x_2 = 0.072$ m, $y_1 = 0.052$ m, $h_2 = 0.037$ m, $\alpha_1 = \alpha_2 = 514\ W/m^2 \cdot °C$, $\lambda = 48\ W/m^2 \cdot °C$, $\Theta_1 = 46°C$, and $\Theta_2 = 62°C$.

Substituting these expressions for temperature into (4), with the cooling water at a temperature of 20°C and a heat-transfer coefficient $\alpha = 4.4\ kW/m^2 \cdot °C$, we find that the losses removed with the water amount to 12.58 kW. Thus 94.5% of all the evolved losses in the insert are removed to the channel, which indicates its high efficiency. On the other hand, the losses removed by the water, as determined experimentally by means of calorimetry [2], amount to 12.8 kW.

Comparison of the theoretical and experimental loss magnitudes confirms the validity of the chosen method.

We can find the temperature field (see Table 1) of the intermediate insert by means of the working formulas (6)-(6a).

NOTATION

$\Theta_1-\Theta_4$	is the temperature of the ambient medium for the four sides of the transverse cross section of the cylinder;
a_1-a_4	are the coefficients of heat transfer for the cylinder surfaces;
r_1, r_2, r	are the inside and outside radii and the instantaneous radial coordinate of the cylinder;
z, z_1	are, respectively, the axial coordinate and the length of the cylinder;
φ	is the angular coordinate;
$P(r, z)$	is the function of the specific losses in the cylinder;
$\eta(z)$	is the Heaviside unit function;
h_1, h_2, r_0, z_0	are the dimensions and coordinates of channel position in the cylinder;
L	is the channel length;
l	is the ratio of channel volume to channel surface;
W	is the intensity of the three-dimensional sink, determined by the magnitude of the losses removed per unit volume of channel;
λ	is the coefficient of thermal conductivity;
x_0, y_0	are the dimensions of the lateral cross section of the insert in the coordinates x and y;
x_1, x_2, y_1, y_2	are the channel coordinates in the lateral cross section of the insert;
β_n, δ_n	are the constant kernels of the integral transform;
α, Θ_0	are, respectively, the coefficient of heat transfer and the temperature of the cooling medium in the internal channel.

LITERATURE CITED

1. E. Wiedemann, The Brown Boveri Review, 53, No. 9 (1966).
2. G. G. Schastlivyi et al., Énergetika i Élektrifikatsiya, No. 2 (1967).